Researchers: Dan DeRose, Kapil Wankhede, Cheshta Chauhan, Venkat Raj Kumar Red Dadireddy

Instructor: Professor Sang Baum Kang

MSF524 Models for Derivatives

09 December 2016

**Pricing and application of a Commodity-linked note**

1. **Executive Summary**

In this paper, tasks related to derivative pricing and financial risk management pricing methods are accomplished. The pricing methods and risk analysis are done using Monte-Carlo simulation and optimal portfolio allocation. Commodity-liked notes are used and are in part motivated by principal protected notes that John Hull’s textbook introduces.

Also, the appropriate payoffs for an option are assessed for different clients with different expectations on price draft and volatility.

1. **Introduction**

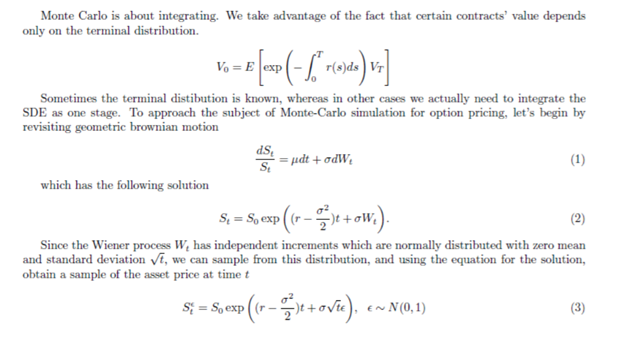
In this paper, we describe the methodology of our pricing models and risk analysis, and discuss the numerical results. We use the control variate technique to improve the convergence speed of Monte-Carlo simulation.

In risk analysis, we use Monte-Carlo simulation and results from our risk-neutral Monte-Carlo pricing model to calculate the expected return and standard deviation that comprise the Sharpe ratio and returns on which we base our recommendations.

We also include recommendation for a client who wants to buy the note for diversification purposes.

Lastly, we propose a business opportunity that combines USD-linked principle protected note, Monte-Carlo simulation, and a market opportunity.

1. **Methodology: Monte-Carlo Simulation**



We use Monte-Carlo simulation to calculate the Returns for our clients and the payoff functions. Black-Schole-Merton Model-related functions are also used in determining what is best for our client.

1. **Numerical Results and Discussion**

Standard Monte Carlo simulation with a reasonable number of simulations (N = 1000), values the note at $393.58, a discount to the $400 face value of the note.

In order to return a sufficiently narrow confidence interval (CI = +/- .01), it is necessary to run almost 53 million simulations. After doing so, the 95% confidence interval for the commodity-linked note is $393.56-$393.58.

1. **Recommendation for Reducing Calculation Speed**

To decrease processing time, it is common in Monte Carlo simulation to utilize a variance reduction technique. Variance reduction is the process of decreasing the variance of a sample Monte Carlo simulation that allows for more precise simulations (i.e. narrower confidence intervals) to be run with a significantly lower amount of repetitions.

Initially, we attempted to decrease computation time by utilizing the antithetic variable technique. The antithetic technique requires identically distributed variables to be horizontally independent. Additionally, the payoff function utilized in the simulation must be monotonically increasing or decreasing. While our commodity-linked note qualifies on the last requirement, the compound structure of the note utilizes correlated variables across the different component underlying securities. These qualify as horizontally dependent and thus the antithetic technique of variation reduction is not appropriate.

Next, we attempted to create a control variate to reduce the variance of the sample Monte Carlo simulation. A control variate is a random variable with a known mean which is exploited to reduce the standard error of an unknown random variable. For the control variate technique to be appropriate, the control variate must have a strong correlation (e.g. strongly positive or negative) to the underlying payoff structure and the expected mean of the control variate must be known.

A basket of European option structures replicating the payoff structure of our commodity-linked note qualifies on both terms: it is strongly correlated to the payoff structure of the commodity-linked note and the mean of a European option is known as European options are easy to price.

To create the correct basket of European options, it was necessary to create identical payoff terms and notional allocations. First, hypothetical strike prices were created that replicate the payoff structure of our commodity-linked note. For example, each commodity is capped at a return of only 25% over the term of the note so a holder of the note is implicitly short a call option 25% above the initial price assumed in the Monte Carlo simulation. Equivalently, the commodity-linked note is principle-protected. That means the holder of the commodity linked note is long an at-the-money put in each commodity. Synthetically, a long put combined with a short call position is called a collar.

Lastly, it is necessary to use the correct notional amount of each commodity when structuring the option collars. Our principle-protected note is equally impacted by all three commodities and is denominated at $400 thus each commodity receives one third of the $400 exposure but scaled to its initial price. This is necessary to find the theoretical expectation of our control variate.

A basket of European option collars is found to be an excellent control variate. The correlation between the payoff of our commodity-linked note and our control variate is 97.72% and the expected price of our control variate is reasonably close to that of our commodity-linked note.

Using the control variate technique, the required number of simulations decreases from 53 million to just over 2 million decreasing simulation time by99% from 237 seconds to just 1.4 seconds. The 95% confidence interval for the control variate simulation was $380.39-380.41.

1. **Recommendation for Customized Deals for Clients under Subject Beliefs**

The recommendations are based on the risk analysis with the components Mean return and Sharpe Ratio.

Mean Return is the mean, of all the likely returns of investments comprising a portfolio therefore an investor would prefer a higher Mean Return. Sharp Ratio is a metric indicating return for every one unit of risk, therefore investor would prefer a higher sharp ratio.

Based on these comparisons, we give our clients the best recommendation with highest Mean Return and a decent Sharpe Ratio [Table 1].

1. **Recommendation for a Client Who Wants to Buy the Note for Diversification Purposes**

Miss Brown is a passive investor of S&P 500 index funds. Currently she has a $10,000,000 investment in S& P 500 Index funds. Miss Brown wants to sell some of her funds and buy our commodity-linked notes. We believe our CLN will significantly diversify Miss Brown’s portfolio and provide her a much more attractive risk/reward tradeoff.

In our risk and return analysis, we first calculated returns, volatilities, and correlations for copper, aluminum, and zinc to calculate the performance of the commodity-linked note. Our assumptions were verified in Microsoft Excel with all modeling done in MATLAB.

[Table 2] Commodity prices are displayed in (closing prices as of Wednesday, November 23).

[Table 3] Drift (mu) is the average annual log returns since 1994.

[Table 4] Volatility is the average annual log variance since 1994.

[Table 5] Correlation is the correlation coefficient of daily prices since 1994.

We assume a 3-year time horizon for our client and use the 3-year Treasury as the risk-free rate. The 3-year yield was 1.39% as of November 23, the date of our simulations.

1. **Risk Analysis and Recommendation**

To make our CLN as attractive as possible to Miss Brown, we thought it would be appropriate to increase the cap in the payoff structure for copper. This increase will allow for a better return and risk trade off because copper has the best return per unit of risk for the three commodities.

**Expected return= 0.0354**

The expected return of the CLN is sufficiently higher than the risk-free rate of return to consider an investment by Miss Brown.

**Sharpe ratio= 0.0944**

Even though the Sharpe ratio of our commodity-linked note is not high on its own, it is important to consider as a diversifier to equities to enhance Miss Browns investment returns.

To determine if Miss Brown will be served well by our CLN, we must find the correlation of the CLN with the S&P 500.

**Correlation between S& P 500 Index and CLN is 0.0298.**

Our CLN has a very low correlation coefficient with a positive expected return. It will be a great diversifier for the Mrs. Brown’s portfolio [Table 6].

Next, we must assign weights to our two assets. Standard investment practice states each weight should be in proportion to the unique risk contributed to the portfolio. Please see [Figure 1]for an illustration of how different weights of CLN and the S&P 500 in the portfolio will result in different amounts of risk and return in the portfolio.

In a 2 asset, risky portfolio, the most efficient allocation of capital should be where return is the highest per unit of risk. Accordingly, the optimal weight of the CLN is 46.34% with the remaining 53.66% invested in the S&P 500. Miss Brown will now have a portfolio with a better risk and reward tradeoff than an undiversified portfolio. With the reduced volatility expected in her portfolio, Miss Brown could even consider levering up this portfolio to match the volatility of the S&P 500 with an even higher expected return! Stuart & Partners would be happy to help!

1. **Proposal of another Business Opportunity**

The world is awash in debt.  Since 2012, the McKinsey Global Institute has published a series of reports on debt and the path of deleveraging ahead. In February, the IMF reported the world had not de-levered at all since the crisis as world debt stood at a record $152tn.

The recent issue of the Fiscal Monitor by the IMF offered the prescription of more deficit spending in hopes that fiscal policy can create policy space for debt to be paid down safely while avoiding any asset shocks. Unfortunately, according to Jaime Caruana of the BIS, it may be too little too late.

Caruana believes calm markets only encourage the build-up of more debt. Cutting edge research by Helene Rey reveals that any respite in advanced markets is matched by capital outflows to emerging markets building up their stocks of debt. “It is the stocks, not the shocks” that matter, according to Caruana, and those stocks are denominated in US dollars. If the USD continues to strengthen, many foreign corporates who borrowed in USD could be in trouble.

Usually an appreciating USD, or depreciating home currency, brings export benefits.

This time, Caruana fears, “the exchange rate takes on a financial amplification role” and any export benefits “may be offset or even reversed by stock valuation effects that operate through the balance sheet.”

That’s why, today, Stuart & Partners is excited to offer a USD-linked Principle Protected Note. The generic version of the USDPPN will use the USD trade-weighted index as its proxy. We will be happy to offer custom versions using any combination of Australian dollars, British pounds, euros, yen, or Mexican pesos. With the recent increase in interest rates, we’re not only able to hedge your exchange rate risk, we can now offer a strong return as well.

Using Monte Carlo Simulation with the appropriate drift and volatility terms derived from the necessary spot and forward yield curves, it will be possible for Stuart & Partners to estimate the expected risk and return of our USDPPN while managing our risk in options.

1. **Conclusion**

We learn about pricing models and risk analysis using Monte-Carlo simulation and optimal portfolio allocation.

For recommendation for customized deals for clients under subject beliefs, we compare the mean returns and the Sharpe ratio to provide the best possible result.

We successfully propose a recommendation for a client who wants to buy the note for diversification purposes.

Lastly, we propose another business opportunity for the client.

1. **A Short 1-page Note for Each Team Member’s Role in the Project**

This project was outlined over the course of several group meet-ups and executed both individually and corporately.

For the MATLAB code, Kapil and Dan worked on parts A and B. Cheshta and Kapil worked on C. Kapil and Dan worked on D.

For the white paper write up, Raj attempted the Executive Summary, Introduction, and Methodology. Dan completed part B. Cheshta completed part C. Kapil completed D. Dan completed part E.

1. **References**

Bodie, Zvi, Alex Kane, and Alan J. Marcus. Investments. Boston, MA: McGraw-Hill Irwin, 2005.

Brandimarte, Paolo. *Numerical Methods in Finance: A MATLAB-based Introduction*. New York: Wiley, 2006.

Caruana, Jaime. "Credit, Commodities and Currencies." Speech, Lecture at the London School of Economics and Political Science, London, February 5, 2016.

Hull, John. *Options, Futures, and Other Derivatives*. Upper Saddle River, NJ: Pearson/Prentice Hall, 2006.

International Monetary Fund. "Debt: Use It Wisely." Fiscal Monitor, October 2016.<http://www.imf.org/external/pubs/ft/fm/2016/02/pdf/fm1602.pdf>.

Jones, Claire. "World Debt Hits $152tn Record, Says IMF." Financial Times, October 5, 2016. Accessed December 1, 2016.<https://www.ft.com/content/6f561ae2-8ad4-11e6-8cb7-e7ada1d123b1>.

McKinsey Global Institute. "Debt and (not Much) Deleveraging." February 2015. <http://www.mckinsey.com/global-themes/employment-and-growth/uneven-progress-on-the-path-to-growth>.

Rey, Hélène. "Dilemma Not Trilemma: The Global Financial Cycle and Monetary Policy Independence." 2015. doi:10.3386/w21162.

1. **Tables and Figures**

[Table 1] Recommendation for Customized Deals for Clients under Subject Beliefs

|  |  |  |  |
| --- | --- | --- | --- |
| **Client 1: P Measure**  Mean Return  0.0424  Mean Excess Return  0.0124  Sharpe ratio  0.0654 | **Recommendation:**  Mrs. Smith will not be happy with the performance of the note. We must offer her better terms. Changing weights and caps is not effective because all three commodities have the same return/risk assumptions. We must change the minimum return to get better results. | **Values after recommendation:**  Mean Return  0.0775  Mean Excess Return  0.0475  Sharpe ratio  0.1488 | **Q measure using new payoff:**  fair value  1.0602e+06 |
| **Client 2: P measure**  Mean Return  0.0281  Mean Excess Return  -0.0019  Sharpe ratio  -0.0106 | **Recommendation:**  Mr. Johnson will not be happy with the performance of the note. We must offer him better terms. Changing weights and caps is not effective because all three commodities have the same return/risk assumptions. We must change the minimum return to get better results. | **Values after recommendation:**  Mean Return  0.0436  Mean Excess Return  0.0136  Sharpe ratio  0.1187 | **Q measure using new payoff:**  fair value  1.0338e+06 |
| **Client 3: P measure** Mean Return  0.0154  Mean Excess Return  -0.0146  Sharpe ratio  -0.1142 | **Recommendation:**  Ms. Williams will not be happy, we need to remove the minimum language to allow her to get full return stream to be realized. Considering correlation is 0, and not worried about all assets taking off at the same time. We must change the minimum return to get better results. | **Values after recommendation:**  Mean Return  0.0568  Mean Excess Return  0.0268  Sharpe ratio  0.0744 | **Q measure using new payoff:**  fair value  1.0522e+06 |
| **Client 4: P measure**  Mean Return  0.0319  Mean Excess Return  0.0019  Sharpe ratio  0.0101 | **Recommendation:**  Mr. Jones has a low Sharpe ratio. We need to change copper cap to realize more of expected return of copper. | **Values after recommendation:**  Mean Return  0.0469  Mean Excess Return  0.0169  Sharpe ratio  0.0599 | **Q measure using new payoff:**  fair value  1.0061e+06 |
| **Client 5: P measure**  Mean Return  0.0424  Mean Excess Return  0.0124  Sharpe ratio  0.0654 | **Recommendation:**  Miss Brown might not be happy as the expectations are too low. We must change cap to realize more upside of zinc. | **Values after recommendation:**  Mean Return  0.0523  Mean Excess Return  0.0223  Sharpe ratio  0.0907 | **Q measure using new payoff:**  fair value  1.0003e+06 |

[Table 2] **Commodity prices**: (closing prices as of Wednesday, November 23)



[Table 3] **Drift** (**mu**) is the average annual log returns since 1994



[Table 4] **Volatility** is the average annual log variance since 1994



[Table 5] **Correlation** is the correlation coefficient of daily prices since 1994



[Table 6]

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Annual Return** | **Annual Std Dev** | **Return / Std Dev** |
| **S&P 500** | **6.98%** | **21.21%** | **0.33** |
| **CLN** | **3.54%** | **22.77%** | **0.16** |
| **Diversified Portfolio** | **5.39%** | **15.75%** | **0.34** |

[Figure 1] Diversification Effects of Commodity Linked Note

